1. Suppose we roll a fair die repeatedly until the sum of all the numbers that appear on the top face is 5. Let X denote the number of times we roll the die. Let F be the cummulative distribution function of X. Compute

   (a) (5 points) $F(1)$,

   (b) (5 points) $F(3)$,

   (c) (5 points) $F(6)$.

2. An box contains 5 white marbles and 2 black marbles. A marble is drawn at random. If the marble is white then it is kept out of the box and a second marble is drawn from the box. If the marble is black then it is replaced back in the box along with another black marble and a second marble is drawn from the box.

   (a) (6 points) What is the probability that both the marbles drawn are white ?

   (b) (9 points) If the second marble drawn is white then what is the probability that the first marble drawn was black ?

3. Let X and Y be independent random variables each geometrically distributed with parameter p.

   (a) (8 points) Find $P(\min(X,Y) = X)$.

   (b) (6 points) Find the distribution of $X + Y$.

   (c) (6 points) Find $P(Y = y | X + Y = z)$.

4. Suppose a random variable X has the following moment generating function.

$$M_X(t) = \frac{1}{4}e^{-4t} + \frac{1}{16}e^{-2t} + \frac{3}{16} + \frac{3}{8}e^{4t} + \frac{1}{8}e^{7t}.$$

Determine

   (a) (4 points) $E[X^2]$ and

   (b) (6 points) $P(\mid X \mid \leq 4)$.

5. (20 points) Each day during the hatching season along the Odisha and Northern Tamil Nadu coast line a Poisson (λ) number of turtle eggs hatch giving birth to young turtles. As these turtles swim into the sea the probability that they will survive each day is p. Assume that number of hatchings on each day and the life of the turtles born are all independent. Let $X_1 = 0$ and for $i \geq 2$, X_i be the total number of turtles alive at sea on the i^{th} morning of the hatching season before the hatchings on the i-th day. Find the distribution of X_n.

6. (20 points) Suppose X is a random variable such that $X \sim \text{Exp}(\,4)$. Let $g : [0, \infty) \to \mathbb{R}$ be given by

$$g(x) = \begin{cases} 0 & x = 0 \\ \frac{1}{x} & x \neq 0 \end{cases}$$

Find the probability density function of $Y = g(X)$.